Vergleich von Hilberträumen

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	Hilbertraum $\mathcal{H}$	**Vektorraum $\mathbb{C}^n$**	**Funktionenraum L^2**
Elemente	$\vert\psi\rangle$ mit $\langle\psi\vert\psi\rangle < \infty$	$\vert v\rangle = \vec{v} = (v_1, \dots, v_n)^T,\ v_i \in \mathbb{C}$	$\vert\psi\rangle = \psi(\vec{r})$ mit $\int \psi^*(\vec{r})\psi(\vec{r})\mathrm{d}^3r < \infty$
Skalarprodukt	$\langle\psi\vert\psi\rangle : \mathcal{H} \times \mathcal{H} \to \mathbb{C}$	$\langle v\vert w\rangle = (\vec{v^*})^T\vec{w} = \sum_{i=1}^n v_i^* w_i :\ \mathbb{C}^n \times \mathbb{C}^n \to \mathbb{C}$	$\langle\phi\vert\psi\rangle = \int \phi^*(\vec{r})\psi(\vec{r})\mathrm{d}^3r$
Elemente des Dualraums	$\langle\psi\vert$	$\langle v\vert = (\vec{v^*})^T = (v_1, \dots, v_n)$	$\psi^*(\vec{r})$
Lineare Operatoren	$\hat{A}$	quadratische Matrizen $A \in \mathbb{C}^{n\times n}$	z.B. Differentialoperatoren $\hat{p} = \frac{\hbar}{i}\nabla$
Adjungierter Operator	$\hat{A}^\dagger$	$\hat{A}^\dagger = (\hat{A}^*)^T$	$\hat{A}^\dagger$
Hermitesche Operatoren	$\hat{A} = \hat{A}^\dagger$	symmetrische Matrizen: $A^* = \hat{A}^T$	$\int(\hat{A}\phi(\vec{r}))^*\psi(\vec{r})\mathrm{d}^3r = \int \phi^*(\vec{r})\hat{A}\psi(\vec{r})\mathrm{d}^3r$
Unitäre Operatoren	$\hat{U}^\dagger = \hat{U}^{-1}$	unitäre (orthogonale) Matrizen	
Erwartungswerte	$\langle\psi\vert \hat{A} \vert\psi\rangle$	$(\vec{v^*})^T A v = \sum_{i,j=1}^n v_i^* A_{ij} v_j$	$\int \psi^*(\vec{r})\hat{A}\psi(\vec{r})\mathrm{d}^3r$
orthonormale Basis	$\{\vert\psi_1\rangle, \vert\psi_2\rangle, \dots\}$	$\{\vec{v}_1, \dots, \vec{v}_n\}$, z.B. $\{\vec{e}_1, \dots, \vec{e}_n\}$	$\{\psi_1(\vec{r}), \psi_2(\vec{r}), \dots\}$
	$\langle\psi_n\vert\psi_m\rangle = \delta_{nm}$	$(\vec{v_n^*})^T\vec{v}_m = \delta_{nm}$	$\int \psi_n^*(\vec{r})\psi_m(\vec{r})\mathrm{d}^3r = \delta_{nm}$
	$\mathbb{H} \ni \vert\psi\rangle = \sum_k c_k \vert\psi_k\rangle$	$\mathbb{C}^n \ni \vec{v} = \sum_k c_k\vec{v}_k$	$L^2 \ni \psi(\vec{r}) = \sum_k c_k\psi_k(\vec{r})$
	$c_k = \langle\psi_k\vert\psi\rangle$	$c_k = (\vec{v_k^*})^T\vec{v}$	$c_k = \int \psi_k^*(\vec{r})\psi(\vec{r})\mathrm{d}^3r$
Dyadisches Produkt	$\vert\psi\rangle\langle\phi\vert$	Matrix $\vec{v}(\vec{w^*})^T = \left(v_i w_j^*\right)_{i,j=1,\dots,n}$	$\vert\psi\rangle\langle\phi\vert\chi\rangle = \psi(\vec{r})\int \phi^*(\vec{r}')\chi(\vec{r}')\mathrm{d}^3r'$
Darstellung der Einheit	$\sum_k \vert\psi_k\rangle\langle\psi_k\vert = \hat{1}$	$\sum_{k=1}^n v_k(v_k^*)^T = E$	$\psi(\vec{r}) = \sum_k \psi_k(\vec{r})\int \psi^*(\vec{r}')\psi(\vec{r}')\mathrm{d}^3r'$
Spur eines Operators	$\mathrm{Sp}(\hat{A}) = \sum_k \langle\psi_k\vert \hat{A} \vert\psi_k\rangle$	$\mathrm{Sp}(A) = \sum_{k=1}^n \vec{e}_k^T A\vec{e}_k = \sum_{k=1}^n A_{kk}$	$\mathrm{Sp}(\hat{A}) = \sum_k \int \psi_k^*(\vec{r})\hat{A}\psi_k(\vec{r})\mathrm{d}^3r$